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A SMALL DISPLACEMENT TORSOR MODEL TO EVALUATE MACHINING ACCURACY IN THE PRESENCE OF LOCATING AND MACHINE GEOMETRIC ERRORS

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Abstract— Precision in Computer Numerical Control (CNC) machining is crucial to manufacturers in their quest to achieve better productivity. During the machining process planning, various factors must be considered, from workpiece design to manufacturing, and finally, to inspection, all of which influence the final quality of the workpiece. In this study, geometric and dimensional errors of fixture's locators, the workpiece datum features' errors in locating a workpiece on a fixture and machine tool geometric errors will be considered in evaluating the accuracy of drilling or milling of a workpiece feature based on the Small Displacement Torsor (SDT) theory. Specifically, the tolerances that are assigned to fixture locators, the workpiece datum feature geometric errors and the machine tool translational and rotational errors are combined in the same model in order to estimate the workpiece machined feature conformity by the mean of the Monte-Carlo tolerance analysis simulation method. This work aims to help fixture designers make better decisions with respect to fixture tolerancing upstream of manufacturing based on a certain predefined confidence level.

Keywords: Machining Fixture, Tool Geometric Error, Drilling and/or Milling Accuracy, Statistical Tolerancing, Geometric Error Evaluation.

I. INTRODUCTION

The inevitable and inherent variations in manufacturing processes make it impossible to obtain the target dimensions of a workpiece in a perfectly repetitive manner. Therefore, certain deviations from the nominal geometry of a workpiece are generally accepted. Geometric Dimensioning and Tolerancing (GD&T) serves to determine and define the limits of acceptance of products in order to meet the functional requirements to which they are subjected. The errors that influence the quality of the final manufactured workpiece are diverse in nature and are not always easy to model or estimate [1].

In the scope of this work, manufacturing fixtures are used to locate, hold and support workpieces in the machine tool reference frame during a specific workpiece manufacturing stage. There are standard/modular fixtures available on the market, and are often used in high-rate production. Nevertheless, in the aerospace and automotive industries, the use of modular fixtures is not always adequate due to the unicity and the complexity of the workpiece to be machined, and so customized fixtures are used. The latter are very expensive to be produced and maintained in good working conditions because the production batch is usually unitary.

One of the main causes of datum-related geometric errors in the workpiece is the deviation of the contact point between a locator and the workpiece surface from the nominal position due to the tolerances assigned to locators. This explains the common assignment of very tight tolerances to the fixture's locators. Usually, a well-known 10-20% industrial rule is applied: the fixture needs to be five to ten times more precise than the part to be machined. This is one of the reasons for the high costs associated with fixture manufacturing. Although such rules may have demonstrated their efficiency, they may be considered exaggerated since the stated precision is not necessarily required. Notwithstanding the fact that the problem has been thoroughly examined from a tolerance analysis perspective [2-5], only Kang [6], to the best of our knowledge, has considered the problem of tolerance allocation in the context of a sensitivity-based algorithm being used to assign tolerances for fixture locators.

The fixture layout design is a very important component of the setup planning process, and is defined as the nominal positioning of fixture components. This is commonly known as "fixture layout optimization", during which, the following three primary factors affecting the final machining accuracy need to be considered by planners:

- Fixturing plan: the locating errors' effects on the workpiece position in relation to the machine tool reference frame.
- Machine tool capability: the machining cutting forces and the machine tool axial motion errors.
- Workpiece rigidity: its response under internal stress release after it is dismounted from the fixture.

The three mentioned sources of errors have attracted a lot of attention in research. In the literature, there are two main approaches used to optimize the fixture layout. The first is based on the rigid body motion theory, and is used to study the locating errors in a statically determinate fixturing widely known as the 3-2-1 locating principle [2, 7]. The second is a FEM (Finite Element Model)-based approach, and is used to predict workpiece behaviour in relation to clamping and machine tool forces [1, 8]. In the work of Vasundara and Padmanaban [9], an exhaustive critical literature review is conducted, and presents the different approaches to optimize a fixture layout. Moreover, extensive literature investigated the effects of the machine geometric errors on the machining accuracy and proposed multiple models to compensate them based on the machine axes' motions [10, 11].

What is remarkable is that most of the literature aim to only model on or at most two sources of errors and rarely simultaneously. It is only in the last recent years that Polini and Corrado [12, 13], established a kinematic chain model that relates the locators' errors, the workpiece form error and the machine tool volumetric error to the geometric errors of a machined feature. The evaluation of the drilling accuracy under singular and combined effects of the three errors was carried out using ANOVA (ANalysis Of Variance).

In this work, the problem of tolerance analysis and synthesis of machining fixtures is reviewed based on the concept of the Small Displacement Torsor, which was originally introduced by Bourdet [14] in order to associate a point cloud to a surface, and which was then used in multiple applications, especially in metrology [15]. Monte-Carlo what-if simulations are used to verify the machining accuracy of datum-related toleranced features under the effect of locating errors (influenced by the tolerances assigned to locators and by the irregularities of the workpiece feature which mates with the fixture) and the rotational and translational errors of the machine tool. A linearized SDT model including the above-mentioned sources of errors is established. The tolerance analysis and synthesis can be done using the proposed approach to a certain confidence level, which means that the designer can analyze/assign tolerances to the fixture locators with a certain prechosen confidence level.

This paper is organized as follows:

In section II, the relationship between the workpiece, the locators and the machine tool is established using the concept of the SDT. Section III presents the geometric error evaluation in accordance with the ASME Y14.5-2018 standard [16]. In section IV, the tolerance synthesis and tolerance analysis of the system fixture-workpiece-machine is treated with respect to the workpiece geometric design specification [17], and section V investigates one simple case study and two real aerospace case studies that were provided by the industrial partner of the research project.

II. Fixture-Workpiece-Machine System Relationship Modeling

The model proposed in this study integrates three sources of errors affecting the final quality of a machined feature, namely, fixture locators' tolerances, the irregularities of the workpiece feature which mates with the fixture, and the machine tool geometric error.

A. Fixture-Workpiece Modeling

An isostatic positioning scheme of the fixture, commonly known as a 3-2-1 locating strategy, is considered. The six degrees of freedom are constrained by six locating pins P_1 to P_6 and three clamps C_1, C_2 and C_3.

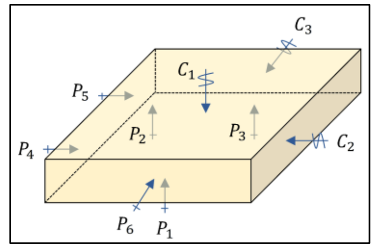

Figure 1. 3-2-1 locating principle

Positioning the workpiece on the fixture can be considered as an assembly of two parts. The six locators P_i ; $i = 1..6$ of the fixture are in contact with their mutual workpiece contact points A_i ; $i = 1..6$. C is a workpiece point that will serve as a transfer point of the torsor vectors. The distance $d_i = d(A_i, P_i)$ is the Euclidian distance between A_i and P_i. When A_i and P_i are in contact, $d_i = 0$ (Fig. 2):

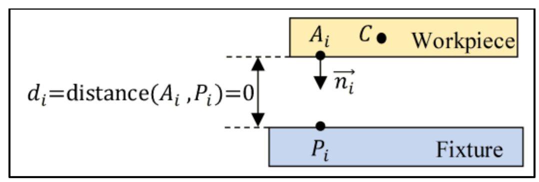

Figure 2. A_i and P_i are in contact

A variation of the position of a locator P_i, whether or not it is combined with a variation of the workpiece point A_i, will result in a relocation of the workpiece. The distance d_i is changed and is no longer null, as shown in Fig. 3, and the workpiece is relocated such that the new distance $d'_i = d(A'_i, P_i')$ becomes null:

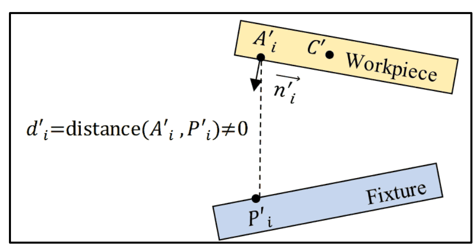

Figure 3. Relocation of the workpiece after deviation

The small displacement torsor concept is used to calculate the new distance d'_i. The displacement of point A is denoted as $\overrightarrow{\mathbf{t_A}} = \overrightarrow{\mathbf{AA'}}$; that of point C as $\overrightarrow{\mathbf{t_C}} = \overrightarrow{\mathbf{CC'}}$, and the rotation of point C as $\overrightarrow{\mathbf{r_C}}$.

Knowing that:

$$\overrightarrow{\mathbf{t_A}} = \overrightarrow{\mathbf{t_C}} + \overrightarrow{\mathbf{AC}} \wedge \overrightarrow{\mathbf{r_C}} \quad (1)$$

And by neglecting the second-order displacements, d' can be written as:

$$d' = -\overrightarrow{\mathbf{t_C}} \cdot \vec{\mathbf{n}} + \overrightarrow{\mathbf{AP}'} \cdot \vec{\mathbf{n}} - \overrightarrow{\mathbf{CP}'} \cdot (\vec{\mathbf{n}} \wedge \overrightarrow{\mathbf{r_C}}) \tag{2}$$

The variation of point P, which is noted $\overrightarrow{\boldsymbol{\delta}\mathbf{P}} = \overrightarrow{\mathbf{PP}'}$, is injected in (2) with $d' = 0$:

$$\overrightarrow{\mathbf{AP}} \cdot \vec{\mathbf{n}} + \overrightarrow{\boldsymbol{\delta}\mathbf{P}} \cdot \vec{\mathbf{n}} = \overrightarrow{\mathbf{t_C}} \cdot \vec{\mathbf{n}} + \overrightarrow{\mathbf{CP}} \cdot (\vec{\mathbf{n}} \wedge \overrightarrow{\mathbf{r_C}}) \tag{3}$$

A matrix writing of (3) is:

$$\overrightarrow{\mathbf{AP}} \cdot \vec{\mathbf{n}} + \overrightarrow{\boldsymbol{\delta}\mathbf{P}} \cdot \vec{\mathbf{n}} = \left[\left(\overrightarrow{\mathbf{CP}} \wedge \vec{\mathbf{n}}\right)^{\mathbf{T}} \quad \vec{\mathbf{n}}^{\mathbf{T}} \right] \begin{bmatrix} \overrightarrow{\mathbf{r_C}} \\ \overrightarrow{\mathbf{t_C}} \end{bmatrix} \tag{4}$$

Finally, the rotation and the translation of point C are:

$$\begin{bmatrix} \overrightarrow{\mathbf{r_C}} \\ \overrightarrow{\mathbf{t_C}} \end{bmatrix} = \left[\left(\overrightarrow{\mathbf{CP}} \wedge \vec{\mathbf{n}}\right)^{\mathbf{T}} \quad \vec{\mathbf{n}}^{\mathbf{T}} \right]^{-1} \cdot \overrightarrow{\boldsymbol{\delta}\mathbf{P}} \cdot \vec{\mathbf{n}} + \left[\left(\overrightarrow{\mathbf{CP}} \wedge \vec{\mathbf{n}}\right)^{\mathbf{T}} \quad \vec{\mathbf{n}}^{\mathbf{T}} \right]^{-1} \cdot \overrightarrow{\mathbf{AP}} \cdot \vec{\mathbf{n}} \tag{5}$$

The relocation therefore depends on two terms. The first one indicates the influence of the variation of the position of a locator P_i and the second represents the influence of the deviation of part datum features on the relocating error. The fixture sensitivity matrix is denoted as:

$$\mathrm{J} = \left[\left(\overrightarrow{\mathbf{CP}} \wedge \vec{\mathbf{n}}\right)^{\mathbf{T}} \quad \vec{\mathbf{n}}^{\mathbf{T}} \right]^{-1} \cdot \begin{bmatrix} \mathrm{n}_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathrm{n_i} \end{bmatrix} \tag{6}$$

Let $F(\alpha_F \quad \beta_F \quad \gamma_F \quad x_F \quad y_F \quad z_F)$ be a point belonging to the workpiece, where $(\alpha_F \quad \beta_F \quad \gamma_F)$ is its orientation vector and $(x_F \quad y_F \quad z_F)$ is its positional vector according to a chosen coordinate system. The new position of point F after relocation is calculated as follows:

$$\begin{cases} \overrightarrow{\mathbf{r_F}} = \overrightarrow{\mathbf{r_C}} \\ \overrightarrow{\mathbf{t_F}} = \overrightarrow{\mathbf{t_C}} + \overrightarrow{\mathbf{FC}} \wedge \overrightarrow{\mathbf{r_C}} \end{cases} \tag{7}$$

B. *Workpiece-Machine Modeling*

The deviation of the feature to be machined is examined as a function of machine geometric errors. The nominal situation and the real situation with the tool geometric error are shown in Fig. 4. C is the center of the part, F is a point of the feature to be machined and T is the center point of the machine tool.

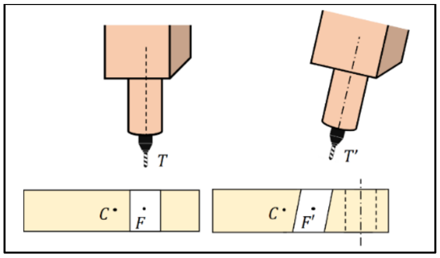

Figure 4. Effect of the machine tool error
Nominal situation (left) – Real situation (right)

The machine tool axis error $(\overrightarrow{\mathbf{r_{T'}}} \quad \overrightarrow{\mathbf{t_{T'}}})$ effect on the machined feature can be written as:

$$\begin{cases} \overrightarrow{\mathbf{r_{F'/C}}} = \overrightarrow{\mathbf{r_{T'}}} \\ \overrightarrow{\mathbf{t_{F'/C}}} = \overrightarrow{\mathbf{t_{T'}}} + \overrightarrow{\mathbf{FT}} \wedge \overrightarrow{\mathbf{r_{T'}}} \end{cases} \tag{8}$$

Equations (7) and (8) can be summed linearly to calculate the deviation of the feature point due to the above three errors.

III. Geometric Error Evaluation of the Toleranced Features

The requirements mentioned in the GD&T control frames related to the workpiece must be met at the end of the machining process. To this end, a passage from GD&T control frames to analytical constraints needs to be realized according to the ASME Y14.5.1M-1994 standard [17].

A. *Hole axis position and orientation*

To control the hole axis position or orientation, the upper and lower points of the nominal axis, $F_1(\overrightarrow{\mathbf{n_{F_1}}} \quad \overrightarrow{\mathbf{Pos_{F_1}}})$ and $F_2(\overrightarrow{\mathbf{n_{F_2}}} \quad \overrightarrow{\mathbf{Pos_{F_2}}})$, are considered in Fig. 5.

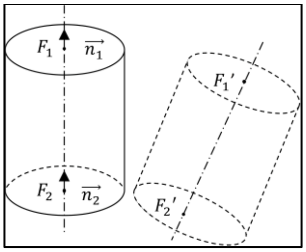

Figure 5. Deviation of the hole axis

After the deviations $\overrightarrow{\boldsymbol{\delta}\mathbf{F_1}}$ and $\overrightarrow{\boldsymbol{\delta}\mathbf{F_2}}$ occur, the two points F_1 and F_2 become F'_1 and F'_2 such that:

$$\begin{aligned} \overrightarrow{\boldsymbol{\delta}\mathbf{F_1}} &= (\delta\alpha_{F_1} \quad \delta\beta_{F_1} \quad \delta\gamma_{F_1} \quad \delta x_{F_1} \quad \delta y_{F_1} \quad \delta z_{F_1}) \\ \overrightarrow{\boldsymbol{\delta}\mathbf{F_2}} &= (\delta\alpha_{F_2} \quad \delta\beta_{F_2} \quad \delta\gamma_{F_2} \quad \delta x_{F_2} \quad \delta y_{F_2} \quad \delta z_{F_2}) \end{aligned} \tag{9}$$

The positional and orientation deviations of the hole axis are calculated as in (10).

$$\begin{aligned} Axis\ Position_{\ error} &= 2 \times max\left(\sqrt{\delta x_{F_1}{}^2 + \delta y_{F_1}{}^2}, \sqrt{\delta x_{F_2}{}^2 + \delta y_{F_2}{}^2}\right) \\ Axis\ Orientation_{\ error} &= \sqrt{\left(\delta x_{F_1} - \delta x_{F_2}\right)^2 + \left(\delta y_{F_1} - \delta y_{F_2}\right)^2} \end{aligned} \tag{10}$$

B. *Planar feature orientation error*

To control the orientation of a planar feature, surface contour points are considered as control points. Under the assumption of rigid body displacement, it is obvious that maximum deviations will occur at these points.

As shown in Fig. 6, the control points F_i are considered; $i = 1..m$ with m is the number of the control points.

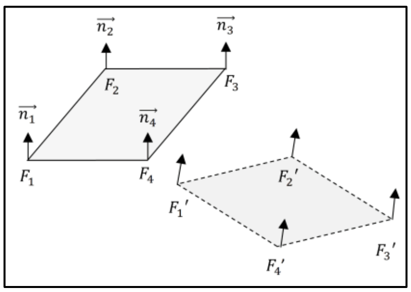

Figure 6. Surface orientation error

After deviations $\overrightarrow{\delta F_i}$, the contour points F_i become F'_i such that:

$$\overrightarrow{\mathbf{\delta F_i}} = (\delta\alpha_{F_i} \quad \delta\beta_{F_i} \quad \delta\gamma_{F_i} \quad \delta x_{F_i} \quad \delta y_{F_i} \quad \delta z_{F_i}) \tag{11}$$

The deviations of the control points along their normal directions are written as:

$$\delta F_i^{\ N} = [\delta x_{F_i} \quad \delta y_{F_i} \quad \delta z_{F_i}] \cdot \overrightarrow{\mathrm{n_i}}^{\mathrm{T}} \tag{12}$$

The surface orientation error deviation is calculated as follows:

$$Surface_{\ error} = max(\delta F_i^{\ N}) - min(\delta F_i^{\ N}) \tag{13}$$

C. *Surface / Line profile error*

For the profile tolerance, the tolerance zone is generated by offsetting each point on the basic profile in a direction normal to the basic profile by a distance equal to half of the profile tolerance. As mentioned before in section III.B, a sample of control points representing the toleranced surface or line profile (Fig. 7) is considered. Then, the deviations along their normal directions $\delta F_i^{\ N}$ are calculated as in (15).

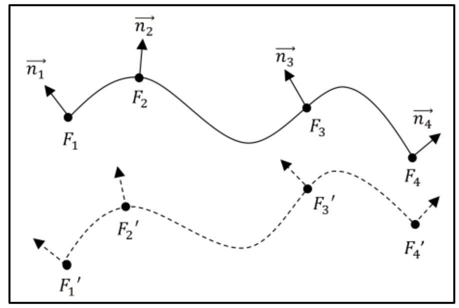

Figure 7. Surface / Line profile control

In the case of the surface (line) profile tolerance, with ASME Y14.5-2018 [16], three control types can be defined as illustrated in Fig. 8.

⌓	.030	A	B	C	Type 1
	.020	A	B	C	Type 2
	.005				Type 3

Figure 8. Types of surface profile tolerances

- Type #1 tolerance to control the exact position.
- Type #2 tolerance to control the orientation.
- Type #3 tolerance to control the form. This type is not affected by the locating errors and is only subject to the machining capabilities.

In the work of Tahan and Levesque [18], new mathematical models for identification and calculation of the profile tolerance considering the process capabilities and the geometric complexity of a part were established.

The type #1 profile error is calculated as follows:

$$Type\ 1_{\ error} = 2 \times \left|\delta F_i^{\ N}\right| \tag{14}$$

The type #2 profile error is calculated as follows:

$$Type\ 2_{\ error} = max(\delta F_i^{\ N}) - min(\delta F_i^{\ N}) \tag{15}$$

IV. Geometric Error Evaluation of the Toleranced Features

Tooling designers always encounter the problem of tolerance assignment and tolerance analysis as they attempt to answer the question: What would be the effect of assigning tolerance T_i to feature F_i on a functional requirement Y?

Fig. 10 illustrates the two scenarios that will be detailed in this study:

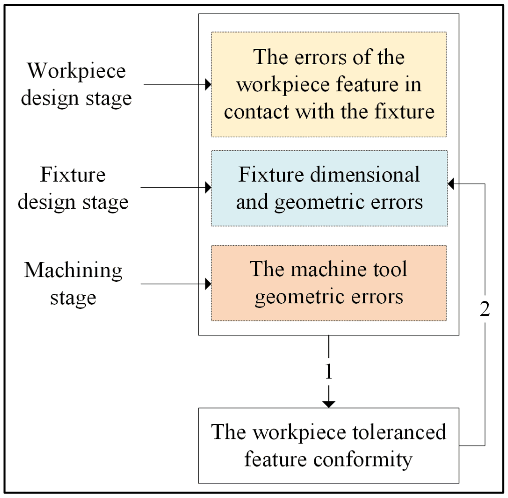

Figure 9. Tolerance analysis scenario (path 1) and tolerance assignment scenario (path 2)

The tolerance analysis process can be laid out as follows: starting from the tolerances assigned to the fixture locators, the errors of the workpiece features that are in contact with the locators and the machine tool errors, what would be their combined effect on the workpiece datum-related toleranced feature that will be machined?

The statistical tolerance analysis methods used in this work require that manufacturing processes be centered and output normal distributions. This correlates to the C_p and C_{pk} values encountered in statistical process control. The tolerance analysis process is stated as follows:

If:

$Tol_i = \pm 3 \times \sigma_{Tol_i}$ is assigned to locator P_i with $i = 1..6$ with the assumption of normality of the variations.

$Tol_{feature} = \pm 3 \times \sigma_{Tol_{feature}}$ is assigned to the workpiece feature which is in contact with the workpiece with the assumption of normality of the variations.

The machine axes errors M_E are following Gaussian distributions such that:

$M_E = (\Delta\alpha_M \quad \Delta\beta_M \quad \Delta\gamma_M \quad \Delta x_M \quad \Delta y_M \quad \Delta z_M)$, where every axis translational and rotational error follows a Gaussian distribution $\Delta_i \sim N(0, \sigma_i)$.

Then, based on (7) and (8), the deviation of any feature point belonging to the workpiece can be estimated. The Monte Carlo simulation method is an efficient way to do this [19]. Statistical moments of the workpiece feature geometric error (such as the expected value, the variance, etc.) are then estimated.

In the tolerance allocation process, the designer tries to assign maximized tolerances to the fixture elements since they are inversely proportional to the cost. It should be mentioned that the tolerances allocated to fixture locators are a function of the fixture layout itself that must be improved before the tolerances are assigned to the locators.

V. Case Studies

In this section, the proposed model is applied to two case studies.

A. *Case study #1*

In this case, a hole should be bored and its axis positioned relative to datums A, B and C in $.010''$, as shown in Fig. 11.

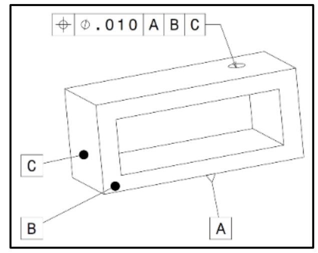

Figure 10. 3D workpiece for case study #

The workpiece is fixtured using the 3-2-1 locating method. Let D_1 and D_2 be two possible layouts of the locators P_i. The locators are more spaced in D_2, which is a common rule used in industry to minimize the effect of the locating errors. The tolerances assigned for the locators are $\pm .004''$ each. The workpiece datum features in contact with the locators should have flatness errors of $.002''$ and machine errors of $\Delta_{rot} \sim N(0, \sigma_{rot} = .0001)$ and $\Delta_{tra} \sim N(0, \sigma_{tra} = .001)$. The results of 10,000 simulated scenarios are shown in Fig. 13 and Fig. 14. The conformity probabilities of the bored hole are 82.46% for layout D_1 and 87.76% for layout D_2.

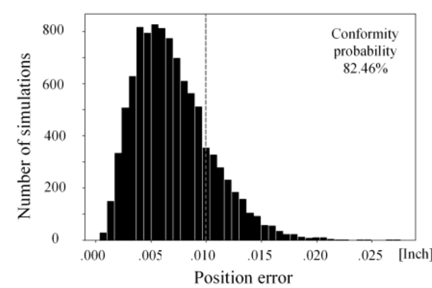

Figure 11. Simulation results for layout D_1

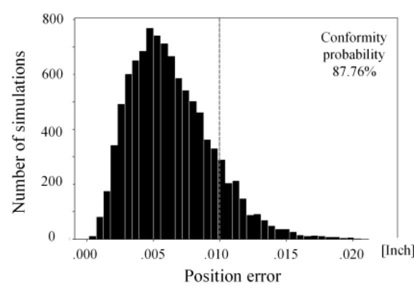

Figure 12. Simulation results for layout D_2

The individual and combined effects on the conformity probability of the hole are shown in Table 2. M denotes the machine errors, P the locators' tolerances, and W the workpiece form errors.

TABLE I. INDIVIDUAL AND COMBINED EFFECTS

Influence	D_1	D_2
P	93,44 %	96,71 %
W	100 %	100 %
M	100 %	99,97 %
$P+W$	88,93 %	92,97 %
$P+M$	86,67 %	91,02 %
$W+M$	99,52 %	99,76 %
$P+W+M$	82,46 %	87,76 %

B. *Case study #2*

The second case study is provided by the project industrial partner. A boring fixture is used to hold the workpiece (Fig .14). The fixturing method can be abstracted as a 3-2-1 locating principle. As mentioned in the illustrated setup, four holes should be bored.

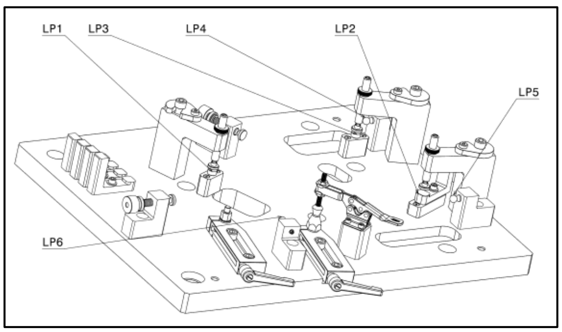

Figure 13. Boring fixture – Case study#2

Fig. 15 presents the tolerance requirements (GD&T). Note that datum targets $A1$, $A2$ and $A3$ are not shown (three contact points between the fixture and the workpiece).

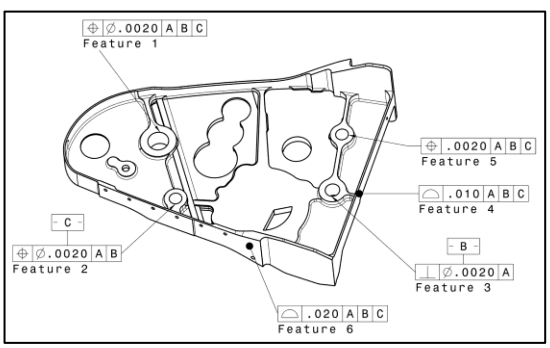

Figure 14. Workpiece GD&T specifications (#2)

The four bores are noted as features 1, 2, 3 and 5. In addition, features 2 and 3 simulate the secondary (B) and tertiary (C) datums, and should be bored in this setup. The surfaces (features 4 and 6) have already been machined in previous stages. The following constraints will be considered in the analyses below:

$$\begin{cases} Feature\ 1: Axis\ Orientation_{\ error} \leq .002 \\ Feature\ 2: Axis\ Orientation_{\ error} \leq .002 \\ Feature\ 3: Axis\ Orientation_{\ error} \leq .002 \\ Feature\ 4: Type1_{\ error} \leq .010 \\ Feature\ 5: Axis\ Orientation_{\ error} \leq .002 \\ Feature\ 6: Type1_{\ error} \leq .020 \end{cases} \qquad (17)$$

The Monte Carlo tolerance synthesis and analysis are done at a 99% confidence level without taking into consideration the machine errors since they are not available.

The tolerance allocation problem is handled with respect to (17). The simulation results using the proposed method are summarized in Table 3.

These values represent the locating point variations allowed to be allocated to the fixture elements intervening in the dimensional chains. An example of such a worst-case scenario tolerance allocation is illustrated in the tolerance chain stack-up of LP2, as shown in Fig. 16, such that:

$$Tol_2 = \pm .006 = \sum_{i=1}^{3} p_i |t_i| \ ; \ p_i = \{-1\ ou\ 1\} \qquad (18)$$

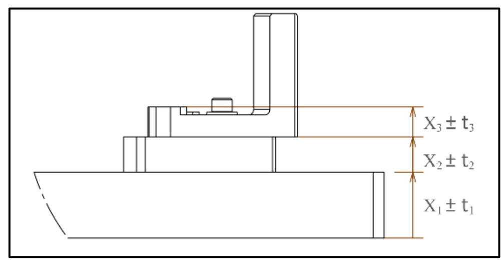

Figure 15. Tolerance chain stack-up of LP2

Using the proposed methodology, the tolerance analysis for each feature can be performed. Here, the process is done for feature 5 at two different sets of locator tolerances, as shown in Table 4.

2500 Monte Carlo iterations are simulated and the orientation tolerance relative to datum A is .002". The simulations results are presented in Fig. 18 and Fig. 19.

TABLE II. SIMULATION RESULTS AT 99% CONFIDENCE LEVEL

$Locator_i$	$Tol_i\ F_1$	$Tol_i\ F_2$	$Tol_i\ F_3$	$Tol_i\ F_4$	$Tol_i\ F_5$	$Tol_i\ F_6$	$Min(Tol_i)$
$Locator_1$	$\pm.009$	$\pm.010$	$\pm.011$	$\pm.008$	$\pm.008$	$\pm.009$	$\pm.008$
$Locator_2$	$\pm.007$	$\pm.008$	$\pm.008$	$\pm.008$	$\pm.006$	$\pm.009$	$\pm.006$
$Locator_3$	$\pm.008$	$\pm.009$	$\pm.009$	$\pm.008$	$\pm.007$	$\pm.009$	$\pm.007$
$Locator_4$	*N/A* [1]	*N/A*	*N/A*	$\pm.005$	*N/A*	$\pm.007$	$\pm.005$
$Locator_5$	*N/A*	*N/A*	*N/A*	$\pm.004$	*N/A*	$\pm.007$	$\pm.004$
$Locator_6$	*N/A*	*N/A*	*N/A*	$\pm.009$	*N/A*	$\pm.008$	$\pm.008$

TABLE III. TWO SETS OF TOLERANCES SIMULATED FOR FEATURE 5

$Locator_i$	Tol_i Set 1	Tol_i Set 2
$Locator_1$	$\pm.008$	$\pm.010$
$Locator_2$	$\pm.006$	$\pm.010$
$Locator_3$	$\pm.007$	$\pm.010$
$Locator_4$	*N/A*	*N/A*
$Locator_5$	*N/A*	*N/A*
$Locator_6$	*N/A*	*N/A*

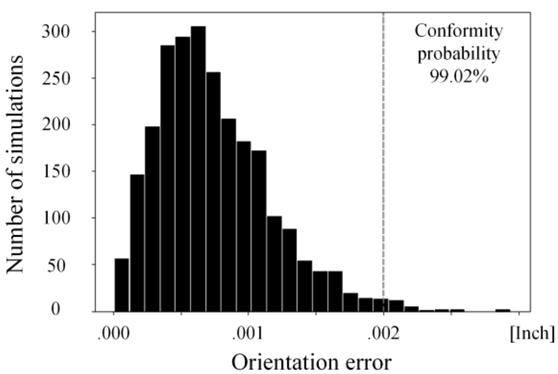

Figure 16. Tolerance analysis with the 1st set of tolerances

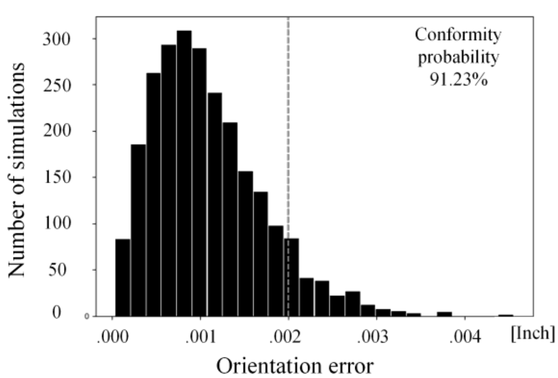

Figure 17. Tolerance analysis with the 2nd set of tolerances

It is obvious that the conformity probability of the hole orientation tolerance specification is reduced when tolerances are less tight. The conformity probability is equal to 99% with the first set of the optimized tolerances (Fig. 18) against 91% with the second set of tolerances (Fig. 19).

VI. CONCLUSION

In this work, a linearized Small Displacement Torsor model combining three sources of errors affecting the final quality of a machined workpiece feature is presented. Based on this model, the accuracy of machining a workpiece feature can be simulated and estimated statistically. This is a tool that helps the fixture designer make better decisions during the fixture design phase by:

- Judging the robustness of a chosen fixture layout.
- Relieving the tight tolerances allocated to fixture elements.
- Analyzing the individual and combined effects of the sources of errors included in the model.

The model is based on the concept of rigid body displacement. For this reason, the effects of the flexibility of the workpiece and the fixture can be ignored.

Furthermore, the translational and rotational errors of the CNC machine could involve tracking its deviations to gather sufficient data in order to estimate these errors. Moreover, depending on the CNC machine type (3, 4 or 5 axes), it is recommended to adapt the model for each one. This is a question that could be explored in a further study.

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